StudyIB.net - Logic

Checklist

Use this space to keep track of your progress with this subtopic. Print and file this document together with those from different sub-topics in a file for quick reference.

Task	Complete (tick or cross)	Traffic Light (Red, amber or green)
Watch the video tutorials		
Check you know your calculators skills		
Review the slides		
Review/annotate the flashcards		
Complete the quiz		
Complete the exam style questions		
Check your solutions against the solution videos		
Review any remaining areas you need to.		

Revision cards

Logic

The Language of logic

Logic Propositions and connectives

a - John plays football

b - John plays hockey

Statements that can be judged as either **true** or **false**

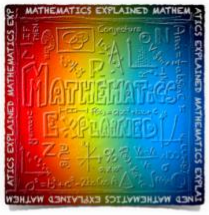
They can be joined together to make **Compound statements**

Statement	Meaning	Symbols
John does **NOT** play football	**NEGATION**	$\neg a$
John plays football **AND** John plays hockey	**CONJUNCTION** - He definitely plays both sports	$a \wedge b$
John plays football **OR** John plays hockey	**DISJUNCTION** - He definitely plays one **OR** the other **OR** both	$a \vee b$
John plays football **OR** John plays hockey BUT **NOT** both	**EXCLUSIVE DISJUNCTION** - He definitely plays either one or the other or **NOT** both	$a \veebar b$
IF John plays football **THEN** John plays hockey	**IMPLICATION** - playing football implies he must play Hockey as well.	$a \Rightarrow b$

Logic Revision cards

Standard Results (Formula Booklet)

p	q	$\neg p$	$p \vee q$	$p \wedge q$	$p \veebar q$	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	F	T	T	F	T	T
T	F	F	T	F	T	F	F
F	T	T	T	F	T	T	F
F	F	T	F	F	F	T	T



Logic

Contradiction, Tautology and Validity

ONLY ever comparing **2 truth** values to deduce a **3rd**

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee q$	$\neg(p \vee q)$	$[\neg(p \wedge q)] \vee q$	$[\neg(p \vee q)] \wedge p$
T	T	T	F	T	F	T	F
T	F	F	T	T	F	T	F
F	T	F	T	T	F	T	F
F	F	F	T	F	T	T	F

Tautology **Contradiction**

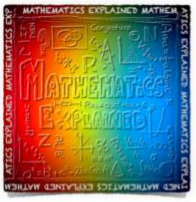
Tautology when it is always **TRUE,** regardless of the truth vales of the initial propositions

Contradiction when it is always **FALSE** regardless of the truth values of the initial propositions

p	q	$p \Rightarrow q$	$\neg q$	$(p \Rightarrow q) \wedge \neg q$	$\neg p$	$[(p \Rightarrow q) \wedge (\neg q)] \Rightarrow (\neg p)$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

If a compound logic statement involves **IMPLICATION** then it is considered an **ARGUMENT**

Because the statement is a **Tautology** we can consider it a **VALID ARGUMENT**



Logic

Equivalence

These statements are **NOT logically equivalent**

$(\neg p \vee \neg q)$

$\neg(p \vee q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(p \vee q)$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	T

NOT logically equivalent because their **truth values** are **different**

Using symbols to express logic equivalence

$(\neg p \wedge \neg q) \Leftrightarrow \neg(p \vee q)$

These statements are **logically equivalent**

$\neg(p \vee q)$

$(\neg p \wedge \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \vee q$	$\neg(p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

Logically equivalent because their **truth values** are the **same**

Logic

Converse, Inverse, contrapositive

REMEMBER these Not in your formula booklet

p - I study hard

q - I do well in my exams

The **Statement**	$p \Rightarrow q$	**IF** I study hard **THEN** I do well in my exams
The **Converse**	$q \Rightarrow p$	**IF** I do well in my exams **THEN** I study hard
The **Inverse**	$\neg p \Rightarrow \neg q$	**IF** I do **NOT** study hard **THEN** I do **NOT** do well in my exams
The **Contrapositive**	$\neg q \Rightarrow \neg p$	**IF** I do **NOT** do well in my exams **THEN** I do **NOT** study hard

Exam Style Questions

Complete these questions on paper and then check your solutions against the video solutions on the website.

Question 1

Consider the following propositions

p – the water level is high, q – it is raining, r - the levee is going to break

a) Write a sentence in words for the following logic statement

$$(p \wedge q) \Rightarrow r$$

b) Write following sentence using logic notation

If the water level is not high or it is not raining then the levee is not going to break

c) What is the contra positive of the statement in part a)? Give your answer using logic notation

Working……

(a) _____

(b) _____

(c) _____

(6marks)

Question 2

Consider the propositions a and b

a – Lions are at the top of the food chain b – Monkeys are wise

a) Write the following in words

i)

$a \vee \neg b$

ii)

$\neg a \Rightarrow b$

b) Complete the truth table below for the two statements above

a	b	$\neg a$	$\neg b$	$a \vee \neg b$	$\neg a \Rightarrow b$
T	T				
T	F				
F	T				
F	F				

c) State, with reasons, whether the two statements below are logically equivalent and give a reason.

$a \vee \neg b$ *and* $\neg a \Rightarrow b$

(a) (i) _____

(ii)_____

(c) _____

(6marks)

Question 3

a) Complete the truth table below to work out the truth values for the following compound statement.

$$(p \vee \neg q) \vee (\neg p \wedge q)$$

p	q	$\neg p$	$\neg q$	$p \vee \neg q$	$\neg p \wedge q$	$(p \vee \neg q) \vee (\neg p \wedge q)$
T	T					
T	F					
F	T					
F	F					

b) What can we say about the compound statement in the final column?

Working……

b) _____